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POSSIBILITY OF RAPID DETERMINATION OF TURBULENCE GENERATION IN A LAMINAR BOUNDARY LAYER

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An engineering method is proposed for determining the initiation of turbulence in a subsonic ($M \leqslant 1$) laminar boundary layer with a heat supply in the presence of a pressure gradient and injection.

A large number of theoretical and experimental works have by now been devoted to the question of the transition of a laminar boundary layer into a turbulent boundary layer. However, this phenomenon (transition from laminar to turbulent boundary layer) is not amenable to rational explanation in every sense. What is needed is a methodological approach which considers both theoretical and empirical aspects of the phenomenon. Possible elements of such an approach, presented below, permit consideration of some of these aspects.

To determine the moment of loss of stability of the laminar boundary layer on the body under consideration, it is necessary to have estimates of the velocity profiles of this layer along the generatrix of the body. However, such estimates are often lacking, or obtaining them proves to be a very complex task. Thus, the stability of an incompressible laminar boundary layer is often determined by using the approximate velocity profile of K. Pohlhausen [1], which adequately describes the solutions of the equation of an incompressible boundary layer in the presence of a pressure gradient:

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$$\frac{u}{u_e} = 2\eta - 2\eta^3 + \eta^4 + \frac{\Lambda}{6}(\eta - 3\eta^2 + 3\eta^3 - \eta^4). \tag{1}$$

The point of loss of stability of the laminar boundary layer (the beginning of turbulence) depends heavily on the first form parameter Λ. The dependences of Re_{cr} and $Re_{\theta t}$ are given in several works. For example, $Re_{cr} = f(\Lambda)$ for an incompressible boundary layer is given in [1], while $Re_{\theta t} = f(\Lambda)$ for a subsonic compressible boundary layer is given in [2]. For rapid determination of the point of loss of stability of a laminar boundary layer with simultaneous injection and and pressure gradient, propose that Λ_{ef} be introduced as follows.

We differentiate Eq. (1) with respect to η:

$$\frac{u_\eta}{u_e} = 2 - 6\eta^2 + 4\eta^3 + \frac{\Lambda}{6}(1 - 6\eta + 9\eta^2 - 4\eta^3). \tag{2}$$

At the wall ($\eta = 0$) we have

$$\frac{u_{\eta w}}{u_e} = 2 + \frac{\Lambda}{6}, \tag{3}$$

where

$$u_{\eta w} = u_{yw}\delta. \tag{4}$$

With injection, u_{yw} decreases. This decrease is accounted for by multiplying u_{yw} in the absence of injection by the coefficient Ψ [3]. We then obtain

$$u_{\eta w} = u_{yw}\delta\Psi = u_{\eta w1}\Psi, \tag{5}$$

where $u_{\eta w1}$ is the derivative of velocity with respect to η in the absence of injection. There is a great deal of published data on the dependence of Ψ on the amount of injection, the composition of the injected gas, and the pressure gradient (see, e.g., [1, 3, 4]).

We substitute the value of $u_{\eta w}$ from Eq. (5) into Eq. (3)

$$\frac{\Psi u_{\eta w1}}{u_e} = 2 + \frac{\Lambda}{6}. \tag{6}$$

It follows from Eq. (6) that

$$\Lambda = \Lambda_{ef} = \left(\frac{u_{\eta w1}\Psi}{u_e} - 2\right)6, \tag{7}$$

where Λ_{ef} is an expression for Λ which considers the simultaneous effect of injection and a pressure gradient. When $\Psi = 1.0$ (no injection):

$$\Lambda = \Lambda_{ef} = -\frac{dp}{dx}\frac{\delta^2}{\mu u_e}; \tag{8}$$

in this case

$$\frac{u_{\eta w1}}{u_e} = 2 - \frac{1}{6}\frac{dp}{dx}\frac{\delta^2}{\mu u_e}. \tag{9}$$

We insert $u_{\eta w1}/u_e$ from Eq. (9) into Eq. (7)

$$\Lambda_{ef} = \left(-\frac{dp}{dx}\frac{\delta^2}{\mu u_e} + 12\right)\Psi - 12. \tag{10}$$

Using the dependence of Re_{cr} on Λ from [1], we calculated Re_{cr} for a plate ($dp/dx = 0$) with different values of suction (or negative injection). Here, instead of Λ in the function $Re_{cr} = f(\Lambda)$, we inserted the value of Λ_{ef}. Comparison of our estimates of Re_{cr} with results calculated by A. Ulrich (results presented in [1]) for exact velocity profiles in accordance with stability theory showed satisfactory agreement within a fairly broad range of suction. For ζ = 0, 0.005, 0.02, and 0.08, $Re_{\delta * cr}$ was equal to 575, 1120, 1820, and 3940 according to Ulrich and 575, 1100, 1850, and 4780 according to our results.

Equation (10) pertains to the boundary layer of an incompressible liquid.

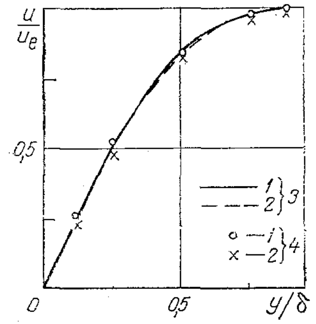

Fig. 1

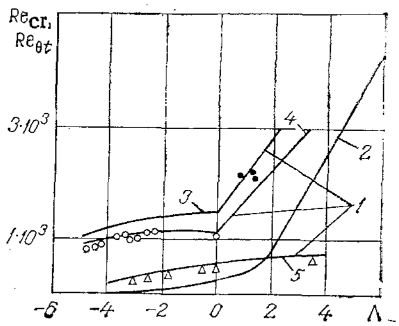

Fig. 2

Fig. 1. Comparison of velocity profiles on a plate obtained from an exact calculation and Eq. (15): 1) $T_0/T_e = 1.2$; 2) $T_0/T_e = 2$; 3) exact calculation, Pr = 1.0, $T_w = T_e$; 4) approximate formula (15), Pr = 1.0, $T_w = T_e$.

Fig. 2. Effect of the form parameter on the numbers Re_{cr} and $Re_{\theta t}$: 1) $Re_{\theta t}$ from [2]; 2) Re_{cr} from [1]; 3) T' = 0.1%; 4) 0.3; 5) 1.25%.

Let us return to examination of Eq. (1). The coefficients in this equation were obtained on the basis of the requirement of the satisfaction of the boundary conditions for problems of an incompressible boundary layer [1]:

$$y = 0;\; u = 0;\; \nu \frac{\partial^2 u}{\partial y^2} = \frac{1}{\rho_e} \frac{dp}{dx} = -u_e \frac{du_e}{dx}, \tag{11}$$

$$y = \delta;\; u = u_e;\; \frac{\partial u}{\partial y} = 0;\; \frac{\partial^2 u}{\partial y^2} = 0. \tag{12}$$

We will examine an approximation of the velocity profile of the equation of a compressible laminar boundary layer. The boundary conditions in this case will be as follows:

$$y = 0;\; u = 0;\; \frac{\partial}{\partial y}\left[\mu \frac{\partial u}{\partial y}\right]_w = \frac{dp}{dx} = -\rho_e u_e \frac{du_e}{dx}, \tag{13}$$

$$y = \delta;\; u = u_e;\; \frac{\partial u}{\partial y} = 0;\; \frac{\partial}{\partial y}\left[\mu \frac{\partial u}{\partial y}\right] = 0. \tag{14}$$

If the velocity profile is approximated by a fourth-degree polynomial and the coefficients are determined so as to satisfy boundary conditions (13) and (14), we obtain the following expression for u/u_e:

$$\frac{u}{u_e} = \frac{1}{6 - \dfrac{\omega T_{\eta w}}{T_w}}\left[12\eta - 6\frac{\omega T_{\eta w}}{T_w}\eta^2 + \left(-12 + 8\frac{\omega T_{\eta w}}{T_w}\right)\eta^3 + \left(6 - 3\frac{\omega T_{\eta w}}{T_w}\right)\eta^4 + \Lambda_w(\eta - 3\eta^2 + 3\eta^3 - \eta^4)\right]. \tag{15}$$

In deriving Eq. (15), we assumed that $T_{\eta e} \cong 0$. This is valid for Pr numbers differing little from 1.

Equation (15) exaggerates the effect of $\omega T_{\eta w}/T_w$ on the velocity profile for large values of $\omega T_{\eta w}/T_w$. However, the agreement between exact calculations of velocity profiles with calculations performed with Eq. (15) is quite satisfactory when $\omega T_{\eta w}/T_w$ is small. Figure 1 compares calculations of profiles according to the program in [4] with calculations with Eq. (15) when dp/dx = 0. It can be seen from Fig. 1 that the agreement is satisfactory. As $\omega T_{\eta w}/T_w \to 0$, Eq. (15) changes into Eq. (1). The exaggerated effect of $\omega T_{\eta w}/T_w$ on the profile in Eq. (15) can probably be reduced by using a more exact velocity-profile approximation. Equation (15) gives us the following (in the same way that Eq. (10) was obtained from Eq. (1)):

$$\Lambda_{ef} = \left(-\frac{dp}{dx}\frac{\delta^2}{\mu_w u_e} + 12\right)\Psi - 12. \tag{16}$$

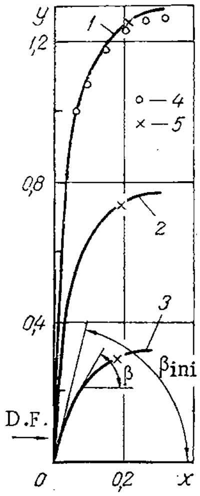

Fig. 3. Contour of the nose of a body on whose surface will be a laminar boundary layer (T_w = 523°K; T_i = 217°K; p_i = 1160 Pa; u_i = 1200 m/sec): 1) β_{ini} = 89.9°, Ψ = 1.0; 2) β_{ini} = 88.7°, Ψ = 1.0; 3) β_{ini} = 77.75°, Ψ = 1.0; 4) β_{ini} = 89.8°, Ψ = 0.3; 5) point where velocity on the boundary of the boundary layer equals the injection velocity (D. F. indicates direction of the flow); x, y, in m.

As an engineering application, let us examine the question of what should be the form of the nose of an axisymmetric body so that laminar flow is maintained on it. As initial data we assign the function $Re_{cr} = f(\Lambda)$, taking it from [1] (Fig. 2). The results of calculation of the form of the body are shown in Fig. 3. The problem was solved as follows: at an initial point (with the coordinates x = 0, y = 0) we assign the angle of inclination of the curve of the contour to the axis of the body $\beta = \beta_{ini}$. Along a straight line with the angle of inclination β, we calculate the pressure p_e, velocity u_e, boundary-layer thickness δ, Λ, Λ_{ef}, and $Re_{\theta run}$. If $Re_{\theta run}$ exceeds Re_{cr}, then the running angle of inclination β decreases by a certain amount dβ. (Here we considered the dependence of Re_{cr} on the temperature ratio T_w/T_e for a compressible boundary layer in accordance with stability theory.) The pressure coefficient was determined in accordance with a modification of Newton's theory. The total pressure on the boundary of the boundary layer was assumed equal to the total pressure at the forward point of the body. The thickness of the boundary layer was calculated from the formula

$$\delta = \frac{5.5x_{ef}}{Re^{0.5}_{x\,ef\,con}}. \tag{17}$$

In Eq. (17) we inserted the coefficient 5.5. This corresponds to a velocity $u = 0.999u_e$ on the boundary of the boundary layer (if $u = 0.99u_e$, then a coefficient of 4.5 must be inserted). The possible error of the determination of 5.5 as the coefficient (even if amounting to 15-20%) has only a slight effect on the final result. The value of x_{ef} was calculated from the formula

$$x_{ef} = \frac{\int\limits_0^x \rho_e u_e \mu_e y_T^2 dx}{\rho_e u_e \mu_e y_T^2}. \tag{18}$$

It can be seen from the results shown in Fig. 3 that the diameter of the body increases with β_{ini}. This is due to the fact that an increase in β_{ini} is accompanied by a decrease in the density of the flow at the boundary of the boundary layer and, thus, the value of $Re_{\theta run}$ at the beginning of the contour. It should be noted that this calculation can be performed only in the subsonic part of the boundary layer, since Re_{cr} begins to depend significantly on M at $M > 1$, and the sign of the dependence of Re_{cr} on Λ changes as well (see, e.g., [5]).

NOTATION

M, Mach number; u, flow velocity; p, pressure; T, temperature; x, y, coordinates; $\eta = y/\delta$, dimensionless coordinates; δ, thickness of the boundary layer; $\Lambda = -\frac{dp}{dx}\frac{\delta^2}{\mu u_e}$, first form parameter; ν, kinematic viscosity; $\mu = BT^{\omega}$, absolute viscosity; B, const; ω, exponent in the formula for absolute viscosity; $T_{con} = (T_w + T_e)\frac{1}{2} + 0.165\frac{\sqrt{Pr}\,u_e^2}{2gC_pA}$, controlling temperature; T_0, stagnation temperature; c_p, specific heat at p = const; Pr, Prandtl number; g, acceleration due to gravity; A, equivalent of heat conversion to mechanical work; θ, momentum thickness; δ^*, displacement thickness; Re, Reynolds number; $Re_{cr} = \left(\frac{u_e\theta}{\nu_e}\right)_{cr}$, Reynolds number of loss of stability of the boundary layer; $Re_x = \frac{u_e\rho_e x}{\mu_e}$; $Re_\theta = \frac{u_e\rho_e\theta}{\mu_e}$; $Re_{\delta^*} = \frac{u_e\rho_e\delta^*}{\mu_e}$; $Re_{x\,con} = \frac{u_e\rho_{con}x}{\mu_e}$; $T_\eta = \frac{dT}{d\eta}$; $u_\eta = \frac{du}{d\eta}$; $u_y = \frac{du}{dy}$; y_T, distance from a surface point of the axisymmetric body to the axis; ρ, density; β, angle of inclination of a plane tangent to the surface of the body to the axis of the body; T', magnitude of turbulent pulsations in the incoming flow, % of u_e; C_f, friction coefficient; $\Psi = \frac{C_f \text{ with injection}}{C_f \text{ without injection}}$; v_w, suction velocity; $\zeta = \left(\frac{v_w}{u_e}\right)^2 Re_x$. Indices: w, surface of body; e, external boundary of boundary layer; con, parameters determined at the controlling temperature; ef, effective parameter; run, running parameter; i, incoming flow; t, beginning of transition; cr, loss of stability of the laminar boundary layer.

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